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Incorporating secondary compression in a settlement analysis

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ABSTRACT The secondary compression of a clay soil is defined as the compression that develops after the induced pore pressure has dissipated. It is a function of shear forces in the very thin, high viscosity layers of absorbed water and shear deformation of these layers. The secondary compression has conceptual inconsistencies regarding its onset and dependence on consolidation time. For example, when the consolidation time, t_{CONS}, for an embankment-loaded soil deposit, calculated by employing the coefficient of secondary compression (C_α) and the vertical coefficient of consolidation (c_v), has been shortened to the c_h-coefficient by the use of wick drains, the secondary compression strain, ε_{2nd}, would be greatly increased were the calculation to calculate the C_α-coefficient from the c_h. Moreover, secondary compression is conventionally calculated employing the average degree of consolidation. Modern computer-assisted calculation can instead consider the time for when the consolidation has developed in a series of strips from the soil layer boundary to the center, making for a smoother transition to secondary compression throughout a soil layer. Secondary compression is but an empirical fitting concept for analysis and design, as demonstrated in a case history of settlement of a test embankment.

Keywords: Immediate compression, consolidation compression, secondary compression, wick drains

Introduction

A foremost of Terzaghi's many contributions to geotechnical engineering is his introducing the principles of effective-stress analysis, compressibility, and consolidation of saturated clay (Terzaghi 1943). Current geotechnical practice applies Terzaghi's approach almost without any additional development, notwithstanding the numerous theses and papers published over the past about 80 years addressing consolidation.

Terzaghi defined the compression as a process initiated by an increase of effective stress comprising a complex process of 'elastic' and non-elastic response, time lag, water flow in restricted-size pores, and viscous creep, separating foundation settlement on three components: immediate compression, consolidation, and secondary compression.

The immediate compression component is a more or less linear elastic deformation with an associated volume reduction of the soil solid particles (compression of the soil skeleton) due to the increase of total stress. ("Immediate" is a subjective function of time—minutes or days even weeks). The immediate compression does not affect the void volume, that is, it occurs without expulsion of water.

The consolidation and secondary compression components are the volume reduction due to reduction of the voids. In a saturated soil, in order for the void volume to reduce, the water must first be expelled—and the pore pressure dissipates in the process. Terzaghi termed this compression occurring as a function of time for pore pressure dissipation "consolidation". The consolidation component is the largest part of the void compression. The secondary compression component is defined as void reduction that continues after the pore pressure has dissipated. It continues at diminishing rate for ever. In a 1946 report, Terzaghi called it "secondary time effect" (Chang 1981).

Fig. 1 shows the typical response of a soil over time after an increase of stress, including the consolidation process—the gradual increase of effective stress, which delays the soil response. Note, the figure is not to scale and, in practice, the difference between the soil structure and consolidation curves can be much other than that shown.

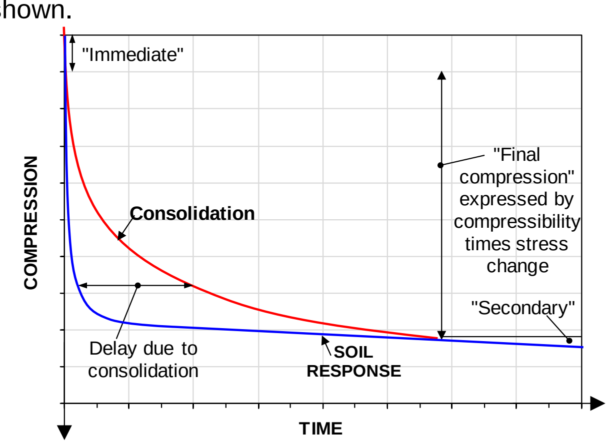

Fig. 1 Compression versus time

The consolidation process is expressed in the Terzaghi Consolidation Theory. The compression is often thought to indicate the settlement of the soil disregarding the immediate and secondary compression components.

Terzaghi stated that the secondary compression (expressed in Eq. 1) is neither a function of the applied load (stress increase) itself, of the soil compressibility, nor of any particular parameter determined for the soil, but, simply, governed by the ratio of time counted from start of consolidation and the time needed from start to full dissipation (i.e., the ratio is always larger than unity).

$$(1) \qquad \varepsilon_{2nd} = \frac{C_\alpha}{1+e_0}\,\lg\frac{t_\alpha}{t_{CONS}} = C_\alpha\, s\,\lg\frac{t_\alpha}{t_{CONS}}$$

where ε_{2nd} = secondary compression strain
C_α = coefficient of secondary compression
e_0 = void ratio (initial) = v_v/v_s
s = solids ratio (initial) = v_s/v_t
t_α = time elapsed since start of consolidation ($t_\alpha > t_{CONS}$).
t_{CONS} = time for completing the consolidation process, which is time between start of the consolidation process (placing the load) and, usually, the time for reaching 90 % or 95 % degree of consolidation per Eq. 3.21.

The secondary compression is much smaller than the consolidation compression. Many find it confusing to include it in design analysis of foundation settlement and choose to disregard it. For most foundation design cases, this would be inconsequential. However, sometimes it would not.

Calculation of Settlement

On applying stress to a saturated soil, the response is an immediate compression and a pore pressure increase that about matches the stress increase. The thereafter response is a time-dependent additional compression (consolidation), as the pore water is forced out and the pore pressure gradually dissipates. When all pore pressure has dissipated, the solids and the soil skeleton continue to adjust to the changed internal shear force distribution, resulting in a small compression that continues over a long time: the secondary compression—or time effect.

A 100-% consolidation requires infinite length of time. Therefore, by the usual definition, when 90 to 98 % (usually 95 %) of the excess pore pressures have dissipated, consolidation is considered to be finished and secondary compression starts (In the 1946 report, Terzaghi suggested 80 %). The rational being that, because secondary compression is compression that occurs at no pore pressure change, it does not exist together with excess pore pressures and ongoing consolidation (dissipation), notwithstanding that it, ostensibly, is triggered when a change of effective stress is imposed. The logic is here somewhat unsatisfactory.

Moreover, the magnitude of secondary compression is also assumed to be a function of the length of time for the primary consolidation to finish (again, taken to be between 90 and 98 % and, usually 95 %) and to the ratio of solids to total volume, (expressed as v_s/v_t or $1/(1 + e_0)$). Why the magnitude of the secondary compression, as it gradually increases with time, would be a function of the length of time for the preceding consolidation is not clear. The time for consolidation to finish is a function of the length of the drainage path (i.e., layer thickness). Be the length of the drainage path long or short, one would expect that the duration of the secondary compression occurring after the end of the consolidation should be similarly long or short. Also, consider the significant shortening of the consolidation time if vertical drains are installed, while the magnitude of the consolidation settlement is unaffected. Should the consolidation time shortened by the drains then be used for determining the size of the secondary compression?

Terzaghi and Peck (1948) wrote: *The secondary time effect is probably a consequence of the fact that the compression of a layer of clay is associated with slippage between grains. Since the bond between grains consists of layers of absorbed water with very high viscosity, the resistance of these layers to deformation by shear will delay the compression even if the time lag resulting from the low permeability of the clay were negligible*. I have not seen a better explanation to the phenomenon. I might add that compression in response to an increased stress creates a tension-compression shear-force imbalance in the viscous layers of absorbed water that takes time to balance out. This process would affect the solid particles, but it would also affect the soil structure (subsequent angular changes in the skeleton) and, of course, the associated loss of voids volume means that water is expelled. The process is normally so slow and small that it does not generate any significant excess pore pressure. However, Chang et al. (1971) and Chang (1969; 1981), reported a case comprising settlement of soft marine clay, where large secondary compression introduced pore pressure.

The magnitude of the immediate compression is the compression of the solids without reduction of void volume. It is expressed by a usually linearly elastic E-modulus parameter. The magnitude of the compression due to consolidation is expressed by a compressibility parameter (a Janbu modulus number, *m*, or a *CR* or $C_c e_0$ relation) and its development over time is expressed by a coefficient of consolidation, c_v. The secondary compression is expressed by Eq. 1.

The process is illustrated in Fig. 2, showing settlement versus time after adding stress to a soil layer, comprising immediate compression and three curves: consolidation compression, secondary compression, and total compression. The secondary compression starts when the pore pressure average has dissipated (to between 90 and 98 %, here 95 %). Recognizing that the end of the pore pressure dissipation does not occur simultaneously within a soil layer, the %-value refers to the average within a layer. Note the ensuing unrealistic kink (increased rate of settlement) of the total settlement curve occurring at the start of the secondary compression per Eq.1.

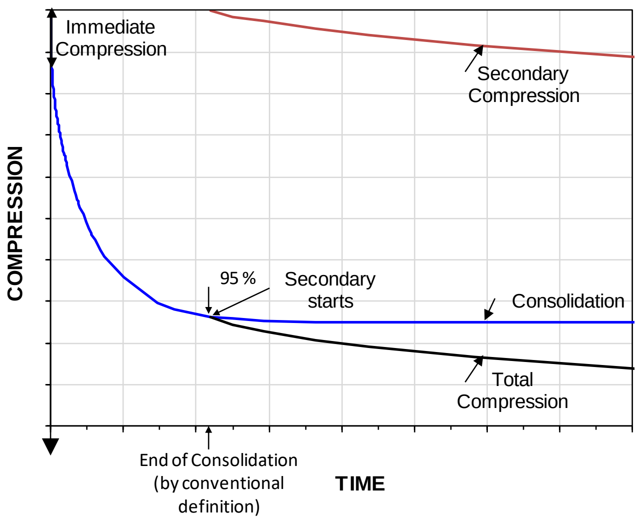

Fig. 2 Time-settlement development with secondary compression related to average consolidation

The dissipation of pore pressure starts almost immediately at the layer boundaries and progresses from there toward the middle of the layer (drainage at both layer boundaries). Fig. 3A shows the typical distribution of pore pressure at different times within a soil layer between free-draining soil layers. Fig. 3B shows the distribution of the time needed for 90 % (here so chosen) of the induced pore pressure to dissipate within each of the series of sublayers and the 90-% average for the entire soil layer. (Graphs are calculated by means of the UniSettle5 software, www.unisoftGS.com, developed by the 2nd author).

As mentioned, the secondary compression is assumed only to start at the end of the consolidation from whence it continues at a diminishing rate. However, the end of consolidation does not occur

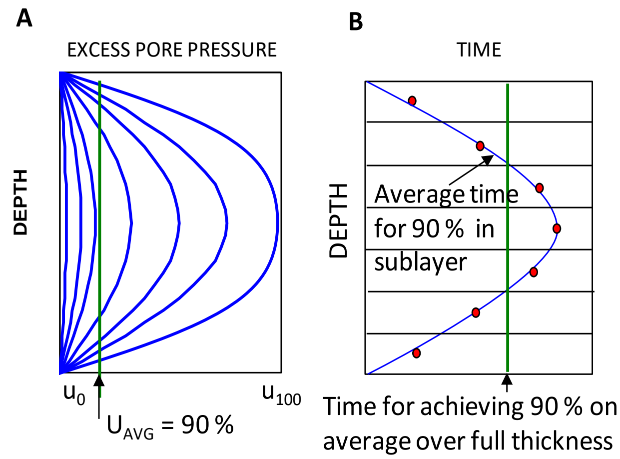

Fig. 3 A. Pore pressure distribution at different consolidation times within a soil layer
B. Average time for achieving 90 % consolidation in layer and sublayers

simultaneously across the layer, but almost immediately at the soil layer boundary and progresses toward the layer center. We can model the soil as composed of a series of sublayers, bands, or strips. Then, as soon as the consolidation has reached the end in a sublayer, by definition, the secondary compression starts in that sublayer. So, the consolidation finishes first in the strip nearest the draining boundary and last in the strip farthest away. The secondary compression, therefore, starts to develop at different times through the soil layer. In the past, due to practical difficulty in considering the different onsets, conventional practice was to assume the secondary compression to start at an average degree of consolidation of the full layer.

Fig. 4 shows the settlement-time development for the same case as used for Fig. 2, when the calculation of secondary compression is applied to the strip or slice of the soil layer whenever the consolidation (95 %) is completed in that strip.

Figs. 2 and 4 show similar long-term values of secondary compression. Intentionally so, which required that the coefficient of secondary compression be set much smaller, by almost an order of magnitude, for the average approach to appear about equal to the varied approach.

Referring to the mentioned fact that the end of consolidation progresses inward from the drainage boundaries, and, therefore, consolidation ends early near the layer boundaries, some simply apply the relation for secondary compression to start at Time 0. This results in a curve without the mentioned unrealistic "kink", but to fit an analysis to observations, applying one or the other method, then necessitates very different input in regard to the chosen coefficient of the consolidation parameter, c_v.

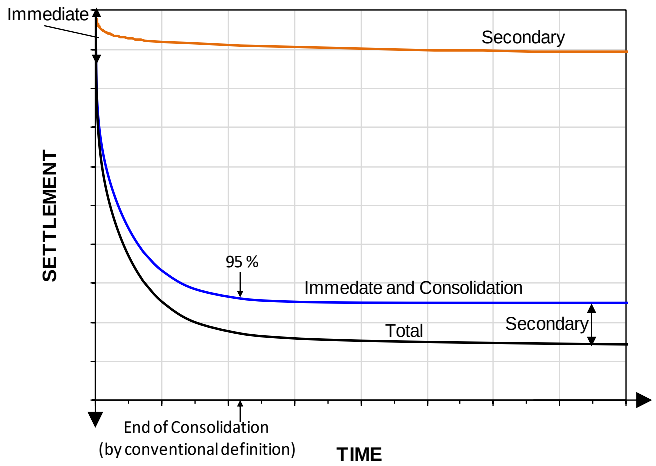

Fig. 4 Time-settlement development with secondary compression related to actual consolidation of sublayers (strips)

Now, how to calculate the secondary compression for a case where the soil consolidation has been accelerated by means of vertical drains? The much shortened drainage path and the fact that horizontal flow is usually faster than vertical mean that the time for full consolidation is much shortened by the use of vertical drains. Should that shorter time be used in calculating the secondary compression? If so, the magnitude of secondary compression would be greatly increased compared to if no vertical drains are used. Would the soil really pay attention to what went on before the full consolidation occurred; before it could start developing secondary compression? The answer is "No". Secondary compression is merely an adjustment to observations, a fudge concept. Still, the concept provides an empirical way to account for the fact that settlement continues after full consolidation has been reached.

In engineering practice, it appears more practical to always calculate secondary compression based on vertical drainage, i.e., the c_v-relation, even if vertical drains have been installed at a site. To illustrate, assume that wick drains were installed a site that develops settlement identical to that shown in Fig. 4. The consolidation process now relies on the horizontal coefficient of consolidation, c_h, usually stated to be four times larger than the vertical coefficient. Fig. 5 shows the calculation results for this change. The figure is a repeat of Fig. 4 with a widened time scale and with the total settlement of the drain-treated site added. Note that the secondary compression is assumed to be equal for the two cases. That is, it is still calculated based on the c_v-coefficient and consolidation by vertical drainage. Were the horizontal drainage and the c_h-coefficient used instead, the secondary coefficient, C_α, would have to be drastically

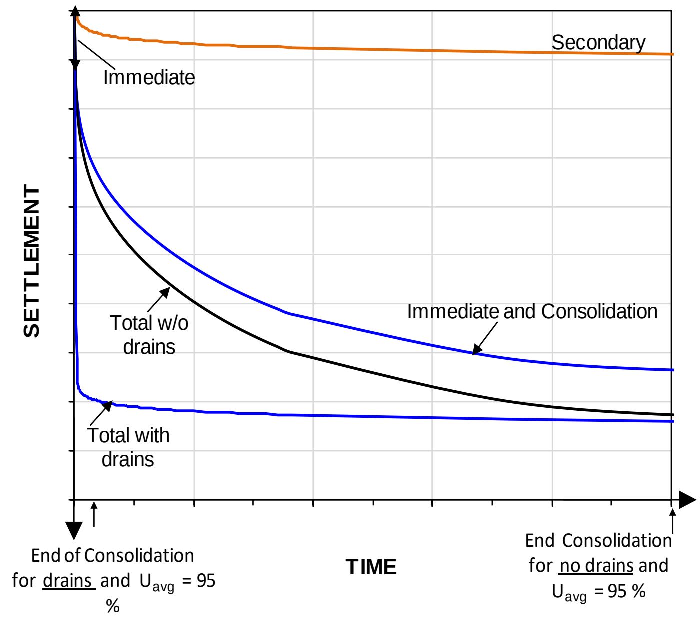

Fig. 5 Time-settlement development for case without and with vertical drains

reduced to bring the secondary compression into reasonable magnitude, i.e., bring it to conform to that calculated for vertical drainage.

The reason for always calculating the secondary compression by applying vertical drainage even in the case of a horizontal drainage condition is because the three components are calculated separately and then added to produce the total settlement over time. Assume that we, instead, would be able to calculate, by some relation or other, the total compression versus time of a soil layer (the "SOIL RESPONSE" shown in Fig. 1) with the voids filled with easily expelled air. It would be independent of consolidation and only respond to the time-dependent compression of the soil (including the immediate and secondary compression). The development of that total compression would be delayed in a saturated soil due to the consolidation, whether by vertical or horizontal drainage, which delay then would be subtracted from the total compression. We are not able to do this calculation, but the idea indicates that the immediate and secondary compression are just concepts to allow engineering estimates. As such they are useful, as indicated by the following back-calculation results of a test embankment in clay treated with wick-drains.

Case History

Lo et al. (2008) reported monitoring records for a 60 m wide road embankment constructed in a swamp area on soft clay improved with wick drains at the Minmi to Beresfield freeway extension site in New South Wales (NSW), west of Newcastle, Australia. The foundation soil and the embankment were instrumented and monitored for about 400 days for excess pore-water pressure and soil stress, and 6.3 years for settlement, respectively.

The soil profile comprised an about 15 m thick, relatively uniform alluvial, normally consolidated, very soft clay layer with a slight crust character to 2 m depth. Below the clay was a highly weathered siltstone with stiff to hard consistency. The water content of the clay was close to the liquid limit and about 100 % at 3.0 m depth reducing to about 60 % at 15 m depth, corresponding to saturated densities of 1,450 and 1,600 kg/m^3, respectively, and void ratios of 2.4 and 1.6, respectively. The natural ground was relatively level. The groundwater table was at 0.5 m depth with an about 0.5 m seasonal fluctuation. Oedometer tests showed a coefficient of consolidation, c_v, in the range of 0.2–0.4 m^2/year and a c_h about twice larger. The compression index, C_c, was in the range of 0.70-0.90. Combined with the void ratio range, the range of the corresponding Janbu modulus number was 7 to 11. No immediate compression modulus or secondary compression index, C_α, were reported.

The test embankment was a 60 m wide part of an about 300 m long, 5.5 m high test embankment next to an existing road. The site instrumentation comprised inclinometers, piezometers, and settlement plates. Wick drains, 100 mm wide, 5 mm thick, were installed to full clay depth in triangular pattern and at a 1.5-m spacing. The embankment was constructed in stages and surcharged in an attempt to reduce post-construction settlement.

Fig. 6 shows the measured and fitted total settlements of the original ground surface at the embankment center with calculated immediate settlement and secondary compression curves. To achieve the fit of simulated to measured settlements, we have simulated and fitted the center settlement in a UniSettle5 analysis using m = 10 (Janbu modulus number) and a horizontal coefficient of consolidation, c_h, of 0.5 m^2/yr. No particular information was available for choosing the input of the E_{ic}-modulus; a 30-MPa value was used to obtain compressions that were reasonable in relation to the measured total settlement. The fit included input of a secondary compression coefficient, C_α, of 0.006.

The calculated total settlement versus time is a function of three interactive curves and several independent soil parameters. Therefore, a fit to the main appearance of the measured settlement can always be produced and no specific parameter can be considered definite.

In designing the later constructed freeway, the Lo et al. (2008) back-calculated parameters were reliably applied to the embankment design ensuring that limits of acceptable settlement would not be exceeded. The fact that the evaluated secondary compression was dubious was of no consequence in this regard.

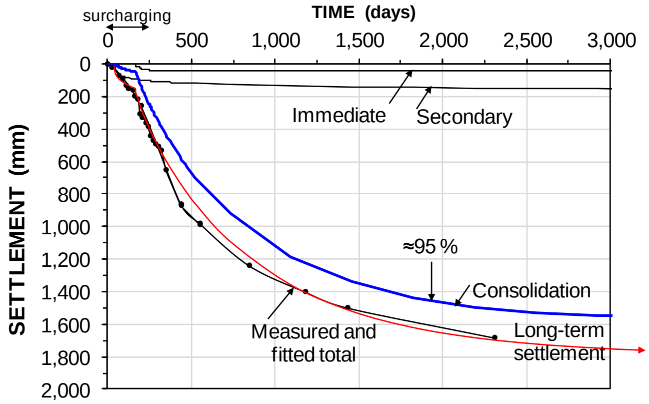

Fig. 6 Monitoring results of embankment in New South Wales (data from Lo et al. 2008)

Summary

The final magnitude of settlement is a function of several factors combined in the "compressibility" of the soil correlated to stress increase, from the compression being directly proportional to stress to non-linear proportionality by some relation or other. The time-dependant compression is quite complex and involves capillary force, particle interaction, viscous response, and many other aspects. There is currently no generally used model for determining it based on soil parameters. Because compression is governed by consolidation, being the slower procedure, the practice applies the Terzaghi principles of pore pressure dissipation and expresses compression as a percentage of that at full consolidation.

The final magnitude is a function of several factors combined in the "compressibility" of the soil correlated to stress increase, from the compression being directly proportional to stress to non-linear proportionality by some relation or other.

Modeling compression versus time of a soil structure set off by an increase of effective stress involves several parameters, such as grain size, water content, soil angularity and flakiness, mineral composition, permeability, grain size distribution, and soil structure chains, micro composition, grain slippage, internal viscosity, and much more.

Consolidation is not the deformation, but a delay of compression. When consolidation is completed, the initiated compression continues. It is desirable that practice develops a method to model the soil compressibility and development over time without having to apply the misconception of settlement being composed of consolidation and secondary compression. The conventional model is just a description that fits observations. The current modeling and lack of a sound theory was questioned by Schmertmann (1983) and his questions are still unanswered.

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